# 3<sup>rd</sup> Nordic Chemistry Olympiad

# **Theoretical Problems**

18<sup>th</sup> July 2018 Oslo, Norway

## Instructions

- · Write your name and country code on all pages.
- The test booklet consists of 6 tasks and 29 pages.
- You have 5 hours for the examination. Do not start until the "START" signal is given.
- Use only the ballpoint pen and the calculator that have been handed out.
- Read the entire task before you begin to solve it, as there may be important information at the end of the text that you need to use to solve the task.
- All results must be written within the boxes. Answers written outside the boxes will not be reviewed and corrected. If you need paper, use the back of the paper sheets.
- Show your calculations in the boxes. You will only get max points if you show your calculations.
- When you have finished the exam, you must pass the test booklet to the person in charge.
- You must stop writing when the "STOP" signal is given.
- Stay in your place until you are allowed to leave the room.

#### **Constants and Formulae**

Avogadro's constant,  $N_A = 6.0221 \times 10^{23} \text{ mol}^{-1}$ 

Boltzmann constant,  $k_B = 1.3807 \times 10^{-23} \text{ J K}^{-1}$ 

Universal gas constant,  $R = 8.3145 \text{ J K}^{-1} \text{ mol}^{-1} = 0.08205 \text{ atm L K}^{-1} \text{ mol}^{-1}$ 

Speed of light,  $c = 2.9979 \times 10^8 \text{ m s}^{-1}$ 

Planck's constant,  $h = 6.6261 \times 10^{-34} \text{ J s}$ 

Faraday constant,  $F = 9.64853399 \times 10^4 \text{ C}$ 

Mass of electron,  $m_e = 9.10938215 \times 10^{-31} \text{ kg}$ 

Standard pressure, P = 1 bar =  $10^5$  Pa

Atmospheric pressure,  $P_{\text{atm}} = 1.01325 \times 10^5 \text{ Pa} = 760 \text{ mmHg} = 760 \text{ torr}$ 

Zero of the Celsius scale, 273.15 K

1 picometer (pm) =  $10^{-12}$  m; 1 Å =  $10^{-10}$  m; nanometer (nm) =  $10^{-9}$  m

 $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ 

1 amu =  $1.66053904 \times 10^{-27} \text{ kg}$ 

Ideal gas equation: PV = nRT

H = U - PVEnthalpy:

Gibbs free energy: G = H - TS

 $\Delta G = \Delta G^{\circ} + RT \ln Q$ 

 $\Delta G^{o} = -RT \ln K = -nFE^{o}_{coll}$ 

 $\Delta S = \frac{q_{rev}}{T}$ , where  $q_{rev}$  is heat for the reversible process Entropy change:

 $\Delta S = nR \ln \frac{V_2}{V_1}$  (for isothermal expansion of an ideal gas)

Nernst equation:  $E = E^O + \frac{RT}{nF} \ln \frac{C_{OX}}{C_{red}}$ 

Energy of a photon:  $E = \frac{hc}{r^2}$ 

Lambert-Beer law:  $A = \log \frac{I_0}{r} = \varepsilon bC$ 

Integrated rate law

 $[A] = [A]_0 - kt$ Zero order

First order  $\ln [A] = \ln [A]_0 - kt$ 

iii

 $\frac{1}{[A]} = \frac{1}{[A]_{\circ}} + kt$ Second order

Arrhenius equation

 $k = Ae^{-E_a/RT}$ 

# Periodic table of the elements with relative atomic masses and electronegativities

	0,7	0,7	Cs	_	Rb	<b>37</b> 85,468 <b>38</b>	<u></u>	<b>19</b> 39,098 <b>20</b>	0,9	Na	11 22,990 12	1,0	<u></u>	<b>3</b> 6,941	2.1	1 1,00794
	<b>Ra</b>	0,9	Ba		Sr	87,62	Ca	40,078	1,2	Mg		1,5	Ве	<b>4</b> 9,0121		
	AC (227)	1,0	La	1,2	~	<b>39</b> 88,906	Sc	<b>21</b> 44,956								
	구 주	1,3	<b>H</b> f	1,4	Zr	,224	<u> </u>	<b>22</b> 47,87								
58 140,12 59 Ce 90 232,04 91 Th	Db (202)	1,5	Ta		Νb	906	<u></u>	23 50,942 24								
140,91 Pr 231,04	Sg		<b>X</b>	1,8	Mo	5,94	Cr	51,996								
00 144,24 Nd 92 238,03	<b>Bh</b>	<u>-</u>	Re	1,9	Тс	43	M <sub>n</sub>	<b>25</b> 54,938								
93 (237,05) 94 Np	Hs (203) log	2	<b>Os</b>		Ru	101,07	₁ Fe	<b>26</b> 55,845								
150,36 <b>Sm</b> (244,06)	Mt	2	<b>-</b>		Rh	45	င္ပ	<b>27</b> 58,933 <b>28</b>								
151,96 <b>Eu</b> (243,06)		2,2	<b>Pt</b>	2,2	Pd	106,42	ż Z	58,693								
64 157,25 Gd 96 (247)		2,4	Au	8	Ag	<b>47</b> 107,87	Cu	<b>29</b> 63,546								
65 158,93 66 Tb 97 (247,07) 98 BK		1,9	<b>Hg</b>		С	<b>48</b> 112,41 <b>49</b>	Zn	<b>30</b> 65,39 <b>31</b>								
98 (251,08) 99 Cf		1,8	<b>T</b>		ln		Ga	31 69,723 32	1,5	≥	,982	2,0	₩	5 10,811 6		
Ho 99 (252,08) 100		1,9	<b>Pb</b>		Sn	50 118,71 51	Ge	32 72,61 33	1,8	<u>s</u>	,086	2,5	ဂ	6 12,011 7		
68 167,26 69 Er 100 (257,1) 10		1,9	<b>B</b> :		Sb	51 121,76 <b>52</b>	<b>As</b>	33 74,922 34	2,1	P	15 30,974 16	3,0	z	7 14,007 8		
69 168,93 70 Tm 101 (258,1) 102		2,0	Po	2	Te	127,60	Se	78,96	2,5	တ		<b>3</b> ,5	0	15,999		
Yb   102 (259,1) 103		2,2	At (FIG.)	2	_	53	э <b>Б</b>	35 79,904 36	3,0	Ω	- 1	4,0	п	9 18,998		
162.50 67 164.93 88 167.26 69 168.93 70 173.04 71 174.97 Dy Ho Er Tm Yb Lu  (251,08) 99 (252,08) 100 (257,1) 101 (258,1) 102 (259,1) 103 (262,1) Cf Es Fm Md No Lr			Rn		Xe	<b>54</b> 131,29	주	<b>36</b> 83,80		Ą	<b>18</b> 39,948		N <sub>e</sub>	3 <b>10</b> 20,180	Пe	<b>2</b> 4,0026

# **Problem 1**

## 10 % of total

а	b	С	d	е	f	g	h	Problem 1	%
10	10	10	10	10	10	10	10	80	10

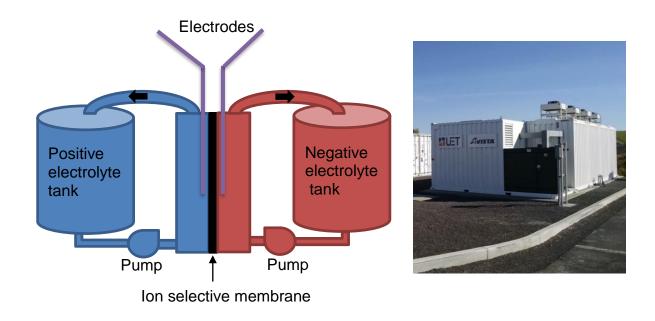


Figure 1, left: Schematic of redox flow battery. Right: 4 MWh containerized vanadium flow battery.

In the hunt for better solutions for energy storage, the redox flow batteries have recently had a resurgence in interest. The battery is comprised of two reaction chambers separated by a membrane with each chamber connected to an electrode and to a tank for storage of electrolyte. The most common of these is the vanadium redox battery.

During discharge the vanadium goes from V(V) to V(IV) in the positive electrolyte tank and from V(II) to V(III) in the negative electrolyte tank. The potential diagram for vanadium at pH = 0 is given below.

$$[\mathsf{VO}_2]^+ \xrightarrow{+0.99} [\mathsf{VO}]^{2+} \xrightarrow{+0.34} \mathsf{V}^{3+} \xrightarrow{-0.26} \mathsf{V}^{2+} \xrightarrow{-1.18} \mathsf{V}$$

a) Write the balanced equations for the anode and cathode reactions during discharge

Anode:  $V^{2+} \rightarrow V^{3+} + e^{-}$ 

Cathode:  $[VO_2]^+ + 2H^+ + e^- \rightarrow [VO]^{2+} + H_2O$ 

b) What will the standard electrode potential of the cell be?

```
E0 = 0.99 + 0.26
E^0 = 1.25 \text{ V}
```

c) Assume that the cathode half cell of the vanadium redox battery is connected to a standard hydrogen half cell ( $E^{\circ} = 0$ ). At what pH will the cathode reaction stop being spontaneous? Assume standard pressure and 25 °C and 1.0 mol/L for all vanadium species.

```
E = E0 - 0.059/n*log(Q)
Q = 1/[H+]^2
E0 = 0.059*2*pH
pH = E0/0.059*2
pH = 8.4
```

A new vanadium redox battery is being built in China (scheduled completion 2019) that can deliver 200 MW for 4 hours.

d) Assume that the voltage stays at E<sup>0</sup> and the battery is 90% discarged at the end. How many liters of 1.0 mol/L battery solution is needed for the cathode?

```
ΔG = -nFE (per mole)
200 000 000J*3600*4/(96 485As/mol*1.25V) = 23.88*10^6mol → Ans/0.9 = 26532621 L = 27 000 000 L
```

Liters of solution: 27 000 000 L

e) What will the actual potential of the battery at 90 % discharge (assume 1.0 mol/l of all constituents at starting point and constant pH = 0). Assume 25 °C.

```
[V^{2+}] = 0.1 \text{ mol/L}
[V^{3+}] = 1.9 \text{ mol / L}
[[VO_2]^+] = 0.1 \text{ mol/L}
[[VO]^2] = 1.9 \text{ mol/L}
E = E0 - 0.059/n*log(Q), n = 1
Q = 1.9*1.9/(0.1*0.1)
E = 1.25 - 0.15 = 1.10
```

f) The electrolyte containing V(V) is made by dissolving  $V_2O_5$  in sulphuric acid. Write the balanced equation for the solvation.

```
V_2O_5 + 2H^+ \rightarrow 2VO_2^+ + H_2O
```

g) There is a leak! Suggest a balanced reaction for what happens when the two electrolytes meet. Assume equal molar ammounts of each electrolyte.

```
VO_2^+ + V^{2+} + H_2O \rightarrow V^{3+} + VO^{2+} + 2OH^- (also accept acidic version)
```

h) What happens if a leak in a fully discharged battery mixes the two electrolytes?

Nothing, the electrolytes are mixed and no reaction occurs.

## **Problem 2**

## 10 % of total

а	b	С	d	е	f	g	h	i	j	k	Problem 2	x %
10	10	10	10	10	10	10	10	10	10	10	110	10

**Rohypnol** is a benzodiazepine used in some countries to treat severe insomnia as well as being used in anesthesia.

As for other hypnotic drugs the use of Rohypnol should be limited to short-term use or on an occasional basis by those with chronic insomnia.

One way to make Rohypnol is illustrated in the scheme below:

a) The first step is to make **A**. It is done by a substitution on the aliphatic carbon. Draw the structure of **A**.

b) Draw the structure of **B** and draw a suggestion for the mechanism

c) Draw the structure of  ${\bf C}$ .

### d) Draw the structure of **D**.

### e) Draw the structure of **E**.

# f) After ozonolysis, ${\bf F}$ containing three benzene rings was obtained. Draw the structure of ${\bf F}$ .

g)  ${f F}$  can be cleaved by hydrolysis with diluted  $H_2SO_4$ . Draw the structures of  ${f G}$  and  ${f H}$ .

OH	F NH <sub>2</sub>
G	Н

h) Draw the structure of I.

### i) Draw the structure of **J**.

### j) Draw the structure of **K**.

# k) Draw the structure of ${\bf L}$ , and draw a resonance structure of ${\bf K}$ that can explain the obtained product.

## **Problem 3**

## 10 % of total

а	b	С	d	е	f	g	Problem 3	%
10	10	10	10	10	10	10	70	10

A DNA (2'-deoxyribonucleic acid) strand is an unbranched polymer of repeating nucleotides. Each nucleotide consists of a heterocyclic, nitrogen-containing base, a five-carbon sugar (ribose) and a phosphate group. The DNA segment shown is (5')AGTC(3') and has a formula mass of 1323,72 g/mol.

a) Assuming that equimolar amounts of the four bases are present in DNA, write the number of H-atoms per P-atoms. Calculate (to 3 significant figures) the theoretical mass percentage of H expected upon elemental analysis of DNA.

Number of H-atoms per P-atom:

Number of atoms: 11,3 H per 1 P

Mass % H:

3,43

The most common hydrogen-bonding patterns are those defined by James D. Watson and Francis Crick in 1953, in which A bonds specifically to T, and G bonds to C. This means that the complementary DNA segment to the segment shown in the picture is (5')GACT(3')

b) One strand of a double-heleical DNA has the sequence (5')GCGCAATATTTC(3'). Write the base sequence of the complementary strand.

c) Draw the structural formula of the two standard DNA base pairs and indicate the hydrogen bonds. The sugar-phosphate backbone can be abbreviated as (-R).

- d) The base composition of DNA generally varies from one species to another, In samples of DNA isolated from two unidentified species of bacteria, X and Y, adenine makes up 32 % and 17 % respectively, of the total bases.
  - I. What relative proportions of adenine, guanine, thymine and cytosine would you expect to find in the two DNA samples?
  - II. One of these species was isolated from a hot spring (64 °C). Which species (X or Y) is most likely the thermophilic bacterium and why?

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X A = T = 32%, G = C = 18%

Y

A = T = 17%, G = C = 33%

II.

The higher the G+C content of a DNA molecule, the higher the melting temperature, because G+C pairing has three hydrogen bonds compared to only two in A+T pairing. Thus, higher temperature is needed to separate the G+C pairs. Species Y, having the DNA with the higher G+C content (66%), most likely is the thermophilic bacterium, its DNA has a higher melting temperature and thus is more stable at the temperature of the hot spring.

- e) The DNA double helix in the human body has a mass of about 10<sup>-18</sup> g per 1000 nucleotide pairs; each base pair extends 3.4 Å.
  - I. Calculate the mass in grams of a double helical DNA molecule stretching from the Earth to the Moon (320000 km).
  - II. Your body contains about 45 gram of DNA. How many times could you stretch your DNA from the Earth to the Moon and back (640000 km)?

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 $3.2 \times 10^8 \text{ m} / 3.4 \times 10^{-10} \text{ m/pair} / 1000$ pairs/segment x  $10^{-18}$  g/segment =  $9.4 \times 10^{-4}$  g II.

 $45 \text{ g} / (2 \times 9,41 \times 10^{-4} \text{ g})$ = 2,4 x 10<sup>4</sup> times

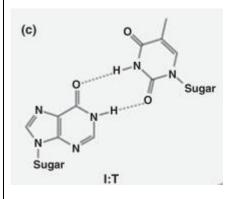
f) The DNA is constantly threatened by internal and external sources that can damage the molecule, including reactive oxygen species, other reactive radicals, as well as various types of ionizing radiation such as UV-radiation. All bases are affected and for adenine, the exocyclic amine group is vulnerable to oxidation. The amine is converted into a ketone. The base that is formed by this oxidation of adenine is called *inosine*. Draw the structure of the base part of *inosine* (you can skip drawing the sugar-phosphate backbone, just indicate the link/position with an R-group)

- g) *Inosine* has different base-pairing properties than adenine, and prefer to base-pair with other bases than the thymine that was originally pairing with the adenine.
  - I. How many of the hydrogen bonds between A and T in the original base pair are disrupted if adenine is oxidized to *inosine*?
  - II. Which of the three remaining bases (A, C or G) do you think form the most stable base pair with *inosine*? Draw the base pair showing the hydrogen bonding pattern.

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2 (both of them)

Note: If someone claims that T might form an alternative base pair with inosine (see figure below), that will be accepted, but the drawing must then be included. Then the answer is zero (0).



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Both solutions are accepted – but the most stable is I:C. The I:A pair will probably be less stable since it contain two large purine bases forming a bulge on the DNA-helix.

# **Problem 4**

## 10 % of total

а	b	С	d	е	f	g	Problem 4	x %
10	10	10	10	10	10	10	70	10

Elemental zinc is a common reductant and chlorine-water a common oxidant in qualitative inorganic analysis. In this task the reaction that occurs when elemental zinc is put into chlorine-water is considered from the thermodynamic point of view. Furthermore, the manufacture and stability of chlorine-water is considered.

The standard reduction potentials are as follows (25.00 °C):

$$Zn^{2+}$$
 (aq) + 2e<sup>-</sup>  $\rightarrow$  Zn (s)  $E^{\circ}$  = -0.7626 V  $Cl_{2}$  (aq) + 2e<sup>-</sup>  $\rightarrow$  2 Cl<sup>-</sup> (aq)  $E^{\circ}$  = +1.3892 V  $Cl_{2}$  (g) + 2e<sup>-</sup>  $\rightarrow$  2Cl<sup>-</sup> (aq)  $E^{\circ}$  = +1.3583 V

The standard third-law entropies are as follows (25.00 °C):

Zn <sup>2+</sup> (aq)	$S^{\circ} = 107.53 \text{ J mol}^{-1} \text{ K}^{-1}$
Zn (s)	$S^{\circ} = 41.6 \text{ J mol}^{-1} \text{ K}^{-1}$
Cl <sub>2</sub> (aq)	$S^{\circ} = 127.6 \text{ J mol}^{-1} \text{ K}^{-1}$
Cl <sub>2</sub> (g)	$S^{\circ} = 222.957 \text{ J mol}^{-1} \text{ K}^{-1}$
Cl <sup>-</sup> (aq)	$S^{\circ} = 56.701 \text{ J mol}^{-1} \text{ K}^{-1}$

The standard state of gases is 1 bar = 100 000 Pa partial pressure.

Let us first consider the reaction of elemental zinc with chlorine-water:

$$Zn (s) + Cl_2 (g) \rightarrow Zn^{2+} (aq) + 2 Cl^{-} (aq)$$

a) Calculate the standard reaction Gibb's energy  $\Delta_r G^\circ$  for the reaction at 25.00 °C.

$$E^{\circ}_{cell} = E^{\circ}_{Cl2} - E^{\circ}_{Zn2+} = 1.3892 - (-0.7626) = 2.1518 \text{ V}$$
 
$$\Delta_{r}G^{\circ} = -zFE^{\circ} = -2 \cdot 96485 \cdot 2.1518 = -415.23 \text{ kJ/mol}$$

b) Calculate the standard reaction enthalpy  $\Delta_r H^\circ$  for the reaction at 25.00 °C

$$\Delta_r S^\circ = 2 \cdot 56.701 + 107.53 - 41.6 - 127.6 = 51.732 \text{ J/(mol·K)}$$

$$\Delta_r G^\circ = \Delta_r H^\circ - T \Delta S^\circ \rightarrow \Delta_r H^\circ = \Delta_r G^\circ + T \Delta_r S^\circ = -399.81 \text{ kJ/mol}$$

c) Calculate the standard cell voltage E° for the reaction at 75.00 °C. You may assume that  $\Delta_r$ H° and  $\Delta_r$ S° do not depend on temperature.

$$\Delta_r G^\circ = \Delta_r H^\circ - T \Delta_r S^\circ = -417.82 \text{ kJ/mol}$$

$$\Delta_r G^\circ = \text{-zFE}^\circ \to \text{E}^\circ = \Delta_r G^\circ / (\text{-zF}) = 2.1652 \text{ V}$$

Let us then consider the dissolution of chlorine into water:

$$\text{Cl}_2\left(g\right) \to \text{Cl}_2\left(aq\right)$$

d) Calculate the equilibrium constant  $k_H$  for the reaction, i.e. the Henry's law constant for chlorine, at 25.00 °C.

```
E^{\circ}_{cell} = E^{\circ}_{Cl2(g)} - E^{\circ}_{Cl2(aq)} = -0.0309 \text{ V} \Delta_{r}G^{\circ} = -zFE = 5962.773 \text{ J/mol} \Delta_{r}G^{\circ} = -RTInK_{H} \rightarrow K_{H} = e^{\wedge}(\Delta_{r}G^{\circ}/RT) = 0.09022
```

e) Calculate the concentration of chlorine in water at equilibrium when a mixture of 0.200 mol chlorine gas and 1.00 L water is sealed into a 2.00 L flask at 25 °C. If you did not find a value for  $k_H$  in d), you can use  $k_H$  = 0.214.

```
\begin{aligned} & n_{Cl2} = [Cl_2]V + n_{Cl2(g)} \\ & K_H = [Cl_2]/p_{Cl2(g)} \rightarrow p_{cl2} = [Cl_2]/K_H \\ & n_{Cl2} = [Cl_2] + 0.4471[Cl_2] = 0.2 \text{ mol} \\ & [Cl_2] = 0.1382 \text{ mol/L} \end{aligned}
```

Finally, let us consider how the concentration of the chlorine-water decreases when the above flask is opened, i.e. how the chlorine-water becomes flat. We consider he effect of opening the flask to be such that all chlorine gas is removed but no chlorine from the solution is neither removed nor entered into the gas phase. No water is removed from the flask, and the temperature remains constant. After resealing the flask a new equilibrium is established, i.e. chlorine is entered from the solution into the gas phase.

f) Calculate the concentration of chlorine in water after the flask has been opened once.

 $0.1382 \text{ mol} = 1.4471 \text{ [Cl}_2\text{]} \rightarrow \text{[Cl}_2\text{]} = 0.0955 \text{ mol/L}$ 

g) How many times of opening the flask causes the concentration of chlorine in water to become less than 10 % of the original?

 $0.2/1.4471^n < 0.02 \rightarrow 10 < 1.4471^n$ log 10 < n log 1.4471  $\rightarrow$  n > 1/log 1.4471) = 6.23

## **Problem 5**

## 10 % of total

а	b	С	d	е	f	g	h	Problem 5	%
10	10	10	10	10	10	10	10	80	10

Carbonic acid, H<sub>2</sub>CO<sub>3</sub> is an elusive species, for long not thought to exist in pure state. Although the pure state is now known to exist under special circumstances, we normally have to consider a range of species including CO<sub>2</sub> (aq), CO<sub>2</sub> (g), HCO<sub>3</sub>-, CO<sub>3</sub><sup>2</sup>-, H<sub>2</sub>O, H<sup>+</sup> and OH<sup>-</sup> when discussing the chemistry of carbonic acid.

a) Draw the Lewis structure for H<sub>2</sub>CO<sub>3</sub>, indicating all valence electrons for all atoms.

b) Theoretically adding carbonic acid to water will create an equilibrium between H<sub>2</sub>CO<sub>3</sub>, CO<sub>2</sub> and H<sub>2</sub>O. Write a balanced equation for the equilibrium between these three species.

$$H_2CO_3$$
 (aq)  $\rightleftharpoons$   $CO_2$  (aq) +  $H_2O$  (I)

The ratio between dissolved carbonic acid and dissolved carbon dioxide in water is given by the hydration equilibrium constant,  $K_H = [H_2CO_3]/[CO_2]$  is  $1.7 \cdot 10^{-3}$  in pure water.

c) What species is the dominating caarbon species only considering the simple equilibrium from task b)?

Since  $K_H \ll 1$ ,  $[CO_2] \gg [H_2CO_3]$ .  $CO_2$  is the dominating species.

The simple equilibrium above does not tell the full story. Carbonic acid is a diprotic acid, with the following dissociation constants:

$$K_{a1} = 2.5 \cdot 10^{-4}$$
  
 $K_{a2} = 4.69 \cdot 10^{-11}$ 

As  $K_{a1} >> K_{a2}$ , it is clear that  $CO_3^{2-}$  can be disregarded for all practical purposes.

d) Disregarding formation of CO<sub>2</sub>, what is the pH of a 0.1 mol/L solution of H<sub>2</sub>CO<sub>3</sub>?

Since Ka1 >> Ka2, the second dissociation can be disregarded

$$H_2CO_3$$
 (aq)  $\rightleftharpoons HCO_3^-$  (aq)  $+ H^+$  (aq)

$$K_{a1} = [HCO_{3}^{-}][H^{+}]/[H_{2}CO_{3}] = 2.5 \cdot 10^{-4} \rightarrow [H^{+}] = 0.005 \rightarrow pH = 2.3$$

Finally, the concentration of dissolved CO<sub>2</sub> is highly dependent on the partial pressure of CO<sub>2</sub> on the surface of the solution, given by Henry's constant as:

$$k_H = \frac{p_{CO_2}}{[CO_2]} = 29.76 \text{ atm/(mol/L)}$$

e) With a typical  $p_{CO_2}$  of  $4 \cdot 10^{-4}$  atm at the surface of the earth, what will be the typical concentration of dissolved CO<sub>2</sub> in a glass of pure water?

 $[CO_2] = 4 \cdot 10^{-4} / 29.76 = 1 \cdot 10^{-5} \text{ mol/L}$ 

With all these expressions at hand, for Ka1, KH, kH, in addition to charge neutrality:

$$[H^+] = [OH^-] + [HCO_3^-] + 2[CO_3^2^-]$$

and the self-ionization of water:

$$K_w = [H^+][OH^-] = 10^{-14}$$

we can paint the full picture of carbonic acid in water.

f) Show that the composition of the solution is fully determined by  $p_{CO_2}$  by finding an expression for [H<sup>+</sup>] that has  $p_{CO_2}$  as the only variable. You may disregard CO<sub>3</sub><sup>2-</sup> completely by saying that [CO<sub>3</sub><sup>2-</sup>] = 0.

$$K_{a1} = \frac{[H^{+}][HCO_{3}^{-}]}{[H_{2}CO_{3}]} (1)$$

$$K_{H} = \frac{[H_{2}CO_{3}]}{[CO_{2}]} (2)$$

$$k_{H} = \frac{p_{CO_{2}}}{[CO_{2}]} (3)$$

$$[H^{+}] = [OH^{-}] + [HCO_{3}^{-}] (4)$$

$$K_{w} = [H^{+}][OH^{-}] (5)$$

Solve everything to leave only [H+] and p<sub>CO2</sub> as variables:

$$\begin{split} [CO_2] &= \frac{p_{CO_2}}{k_H} \quad , \quad [H_2CO_3] = K_H[CO_2] = K_H \frac{p_{CO_2}}{k_H} \\ \\ [OH^-] &= \frac{K_w}{[H^+]} \quad , \quad [HCO_3^-] = [H^+] - \frac{K_w}{[H^+]} \end{split}$$

Now everything can be inserted into (1):

$$K_{a1} = \frac{[H^+]\left([H^+] - \frac{K_w}{[H^+]}\right)}{K_H \frac{p_{CO_2}}{k_H}} = \frac{[H^+]^2 - K_w}{K_H \frac{p_{CO_2}}{k_H}}$$

This simplifies to:

$$[H^{+}] = \sqrt{\frac{K_{a1}K_{H}p_{CO_{2}}}{k_{H}} + K_{w}}$$

g) If you leave a glass of pure water on the bench and let it reach equilibrium for the CO<sub>2</sub> (g), CO<sub>2</sub> (aq) and H<sub>2</sub>CO<sub>3</sub> (aq) system. What will the pH be? If you did not find an expression for [H<sup>+</sup>] in task f), use [H<sup>+</sup>] =  $\left(\frac{K_H K_A}{k_h} p_{CO_2}\right)^{1/2}$ .

$$pH = -\log \sqrt{\frac{K_{a1}K_{H}p_{CO_{2}}}{k_{H}} + K_{w}} = 5.7$$

In a bottle of fizzy drinks, the CO<sub>2</sub> pressure over the liquid is typically 2.5 atm.

h) What is the dominant carbon species in such a drink?

Solving for the different species gives [CO<sub>2</sub>] >> anything else.

[CO<sub>2</sub>] is the dominant species.

## **Problem 6**

### 10 % of total

а	b	С	d	е	Problem 4	%
10	10	10	10	10	50	10

Dinitrogen pentoxide,  $N_2O_5$ , is a reactive molecule used as a nitrant in organic chemistry. When NO reacts with  $NO_2$  and  $O_2$ ,  $N_2O_5$  it is formed according to reaction (1).

$$NO(g) + NO_2(g) + O_2(g) \rightarrow N_2O_5(g)$$
 (1)

The following measurement data was obtained in an experimental series.

[NO] <sub>initial</sub> (mol/dm <sup>3</sup> )	[NO <sub>2</sub> ] <sub>initial</sub> (mol/dm <sup>3</sup> )	[O <sub>2</sub> ] <sub>initial</sub> (mol/dm <sup>3</sup> )	Initial rate (mol/(dm <sup>3</sup> · s))
0.20	0.30	0.20	1.26·10 <sup>-1</sup>
0.10	0.10	0.20	2.10·10 <sup>-2</sup>
0.10	0.10	0.10	2.10·10 <sup>-2</sup>
0.20	0.10	0.10	4.20·10 <sup>-2</sup>

a) Determine the reaction orders for NO, NO<sub>2</sub> and O<sub>2</sub>.

The reaction order for  $O_2$  can be determined from the  $2^{nd}$  and  $3^{rd}$  measurement. Here [NO]<sub>initial</sub> and [NO<sub>2</sub>]<sub>initial</sub> are unchanged but [O<sub>2</sub>]<sub>initial</sub> is varied. Since there is no change in the initial rate, the reaction order for  $O_2$  is 0

The reaction order for NO can now be determined from the  $2^{nd}$  and  $4^{th}$  measurement. Here [NO<sub>2</sub>]<sub>initial</sub> is unchanged but [NO]<sub>initial</sub> is doubled. Since the initial rate also is doubled, the reaction order for NO is 1 ([O<sub>2</sub>]<sub>initial</sub> is different in the two measurements but since the reaction order for O<sub>2</sub> is 0, it doesn't affect the rate)

Finally the reaction order for  $NO_2$  can be determined from the  $1^{st}$  and  $4^{th}$  measurement. Here [NO]<sub>initial</sub> is unchanged but [NO<sub>2</sub>]<sub>initial</sub> is tripled. Since the initial rate also is tripled, the reaction order for  $NO_2$  is 1

b) Calculate the rate constant. Also give the unit of the rate constant.

Using the result from a):

The initial rate =  $k \cdot [NO]_{initial} \cdot [NO_2]_{initial}$ 

Using values from the 1st measurement:

 $0,126 \text{ mol } / (dm \cdot s) = k \cdot 0,30 \text{ mol/dm}^3 \cdot 0,20 \text{ mol/dm}^3 \rightarrow k = 2,1 \text{ dm}^3 / \text{ (mol} \cdot s)$ 

Answer: k = 2,1 unit..... dm<sup>3</sup>/ (mol·s)

The N<sub>2</sub>O<sub>5</sub> molecule decomposes in the gas phase according to the net reaction (2)

$$2N_2O_5 \rightarrow 4NO_2 + O_2 \tag{2}$$

The temperature dependence of the decomposition was studied and the following measurement data was obtained:

T (K)	k (s <sup>-1</sup> )
338	4.9·10 <sup>-3</sup>
318	5.0·10 <sup>-4</sup>
298	3.5·10 <sup>-5</sup>

c) Determine the mean activation energy of the reaction. Give your answer in kJ/mol.

### Solution 1

 $k = A \cdot e^{-E_a/RT}$ 

Use the equation at two temperatures:

$$\begin{cases} k_1 = A \cdot e^{-E_a/RT_1} \\ k_2 = A \cdot e^{-E_a/RT_2} \end{cases} \to \begin{cases} \ln(k_1) = \ln(A) - E_a/RT_1 \\ \ln(k_2) = \ln(A) - E_a/RT_2 \end{cases} \to \ln\left(\frac{k_1}{k_2}\right) = -\frac{E_a}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$$T_1 = 298 \text{ K and } T_2 = 318 \text{ K} \rightarrow E_a = 104.76 \text{ kJ}$$

$$T_1 = 298 \text{ K and } T_2 = 338 \text{ K} \rightarrow E_a = 103.46 \text{ kJ}$$

$$T_1 = 318 \text{ K and } T_2 = 338 \text{ K} \rightarrow E_a = 101.98 \text{ kJ}$$

with an average of  $E_a = 103,4 \text{ kJ}$ 

#### Solution 2

 $\ln(k) = \ln(A) - E_a/RT$ 

Plot ln(k) versus -(1/RT). Use linear regression to obtain slope =  $E_a$  and intercept = ln(A).

You get  $E_a = 103.5 \text{ kJ}$  and  $In(A) = 31.52 \rightarrow A = 4.9 \cdot 10^{13}$ 

Answer:  $E_a = 103,5$  kJ

d) Calculate the amean frequency of the reaction (pre-exponential factor).

#### Solution 1

Using  $k = A \cdot e^{-E_A/RT}$  and the average value of E<sub>A</sub> from c), you can calculate A from the rates at the three temperatures

$$T = 298 \rightarrow A = 4.67 \cdot 10^{13}$$

$$T = 318 \rightarrow A = 4.83 \cdot 10^{13}$$

$$T = 338 \rightarrow A = 4,68 \cdot 10^{13}$$

with an average of 4,7 · 10<sup>13</sup>

#### Solution 2

Having used linear regression in c) you have already obtained  $A = 4.7 \cdot 10^{13}$ 

Answer:  $A = 4.9 \cdot 10^{13}$ 

e) What is the half-life of decomposition of  $N_2O_5$  at 402 K? (If you weren't able to calculate  $E_a$  and A in c) and d), use the values  $E_a = 115$  kJ and  $A = 3.6 \cdot 10^{14}$ . These are however not the correct values)

Since the decomposition has a "half-life" that is independent of concentration (no concentration value was given), it must be  $1^{st}$  order for  $[N_2O_5]$ 

This can also be concluded from the unit, s<sup>-1</sup>, of k in the table

$$\frac{d[N_2O_5]}{dt} = -k[N_2O_5] \to \ln[N_2O_5] = \ln[N_2O_5]_0 - kt \to \ln[N_2O_5] = \ln[N_2O_5]_0 - kt$$

Inserting  $[N_2O_5] = \frac{1}{2}[N_2O_5]_0$  and  $t = t_{\frac{1}{2}}$  gives  $t_{\frac{1}{2}} = \ln 2/k$ 

k at 402 K is calculated from  $k = A \cdot e^{-E_a/RT}$  using the values of E<sub>a</sub> and A obtained in c) and d).

Using  $E_a = 103.5 \text{ kJ}$  and  $A = 4.9 \cdot 10^{13} \text{ gives } k = 1.74 \text{ s}^{-1}$  and  $t_{1/2} = 0.40 \text{ s}$ 

Answer:  $t_{1/2} = 0.40$  sec

The end of the problem set